6.302 Lab 2 Report

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1 Results Summary

	Uncompensated	Gain	Lag	Lead
Prelab Bandwidth ω_b	3200	1250	1256	773
Lab Bandwidth ω_b	3320	1350	1350	1009
Prelab Peak Overshoot P_o	1.67	1.19	1.21	1.22
Lab Peak Overshoot P_o	1.56	1.23	1.27	1
Prelab Time to Peak t_p	0.0015	0.0037	0.0038	0.00075
Lab Time to Peak t_p	0.00144	0.0036	0.0036	∞
Prelab Damped Natural Frequency ω_d	2083	849	817	4178
Lab Damped Natural Frequency ω_d	2310	849	701	–
Prelab Peak Magnitude M_p	4.01	1.21	1.24	1.28
Lab Peak Magnitude M_p	4.32	1.32	1.68	1
Prelab Peak Frequency ω_p	2076	719	706	3665
Lab Peak Frequency ω_p	2224	829	796	0
Prelab Damping Ratio ζ	0.126	0.469	0.450	0.433
Lab Damping Ratio ζ	0.116	0.415	0.379	–
Prelab Natural Frequency ω_n	2110	961	915	4636
Lab Natural Frequency ω_n	2255	1025	943	–

2 Component Values

For the reduced gain compensated system, I used a resister with a value of $R = 4900\Omega$.

For the lag compensator, I used component values of $R = 4900\Omega$ and $C = 2.19\mu F$.

For the lead compensator, I used component values of $R = 4900\Omega$ and $C = 4.38\mu F$.

3 Discussion

The lead compensated system behaved more like a first order system than a second order system; there was no measurable peak overshoot and no indication of frequency dependant magnitude peaking. Consequently, I measured first order metrics for the lead compensator. The lead compensator had a time constant of $\tau = 0.991$ ms. This gives a rise time of 2.2 ms, which is somewhat faster than the lag and reduced gain compensators' 3.6 ms time to peak.

The lead compensator performed very differently than expected. It's measured rise time was better than both its expected value and the measured time to peak of the other compensators. In addition, it showed no ringing or overshoot, which suggests better stability. The reason the lead compensator doesn't match my analysis is that I made a mistake in my analysis. Unlike the reduced gain and lag compensators, the lead compensator didn't easily appear in unity feedback form. In my prelab, I incorrectly calculated the root locus using the open loop gain. If I had instead manipulated the block diagram until it was in unity feedback form, I would have found that one of the zeros in the feedback path would have become a pole in the closed loop system. Since that pole is at a much lower frequency ($\tau = 0.95$ seconds) than the other poles in the closed loop system, it dominates the system behavior, making the closed loop system look like a first order system.

The lag and reduced gain compensators performed very similarly; they both showed reasonably high bandwidth with much smaller magnitude peaking than the uncompensated system. The lag compensator also showed somewhat greater magnitude peaking than the reduced gain compensator, although both had nearly identical DC gain. In contrast, the lead compensator had slightly higher DC gain than either the lag or reduced gain compensator.

In light of my lab results and my new analysis of the lead compensator, I can say that the lead compensator seems to offer the best alternative. It provides greater stability, a fast transient response, and no magnitude peaking. However, it does have slightly lower bandwidth than the other compensators, and hence takes much longer to settle out.

4 Calculation Methods

For the reduced gain and lag compensators, I used these formulas since they were best modeled as second order systems.

$$\zeta = \sqrt{\frac{(\ln P_o - 1)^2}{\pi^2 + (\ln P_o - 1)^2}}$$

$$\omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}}$$

$$\zeta = \sqrt{\frac{1 \pm \sqrt{1 - \frac{1}{M_p^2}}}{2}}$$

$$\omega_n = \frac{\omega_p}{\sqrt{1-2\zeta^2}}$$

For the lead compensator, I used these formulas since it was best modeled as a first order system.

$$t_r = 2.2\tau$$

$$\omega_h = \frac{1}{\tau}$$