6.302 Motor Lab Take Home Quiz

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(a) The assumption that the rise time of I_m is approximately $2.2L/R$ is only valid assuming that the mechanical time constant of the motor is much greater than the electrical time constant of the motor.

This experiment involves driving the motor with a step of voltage. That change in voltage causes a change in the motor current I_m which changes the torque and hence, the angular velocity of the shaft. This increase in angular velocity $\dot{\theta}$ produces a back EMF that counters the input voltage step. However, because the mechanical componants of the motor are much slower to respond than the electrical componants, I_m can undergo a complete first order response to the voltage step input before its effect changes the angular velocity, and hence, the input voltage seen by the motor.

(b) The nonzero steady state error was not expected but is easily explained. Since the motor was in current drive,

$$L(s) = 0.4K_{tach}G_c\frac{\dot{\theta}}{I_m} = 0.4K_{tach}G_c\frac{K_T}{Js}$$

the error function should have been

$$E(s) = \frac{1}{1 + L(s)} = \frac{Js}{Js + 0.4K_{tach}K_T G_c}$$

Using the Final Value Theorum, we see that as time goes to infinity, the error becomes

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = 0$$

The system produces zero steady state error because it includes an integrator in the forward path, which is why I expected to see zero steady state error to a step input. However, this model is flawed since it assumes no friction. If we expand our model to include friction, we see that

$$L(s) = 0.4K_{tach}G_c\frac{K_T}{Js + B}$$

$$E(s) = \frac{Js + B}{Js + B + 0.4K_{tach}K_T G_c}$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{B}{B + 0.4K_{tach}K_T G_c}$$

When our model includes friction, the integrator is replaced with a real axis pole, thus giving nonzero steady state error.

(c) Since this compensator has a pole at $\omega = \frac{10}{\tau}$ and a zero at $\omega = \frac{1}{\tau}$, the zero occurs before the pole. Therefore, this compensator must be a Lead compensator.

The closed loop transfer function for the uncompensated system is

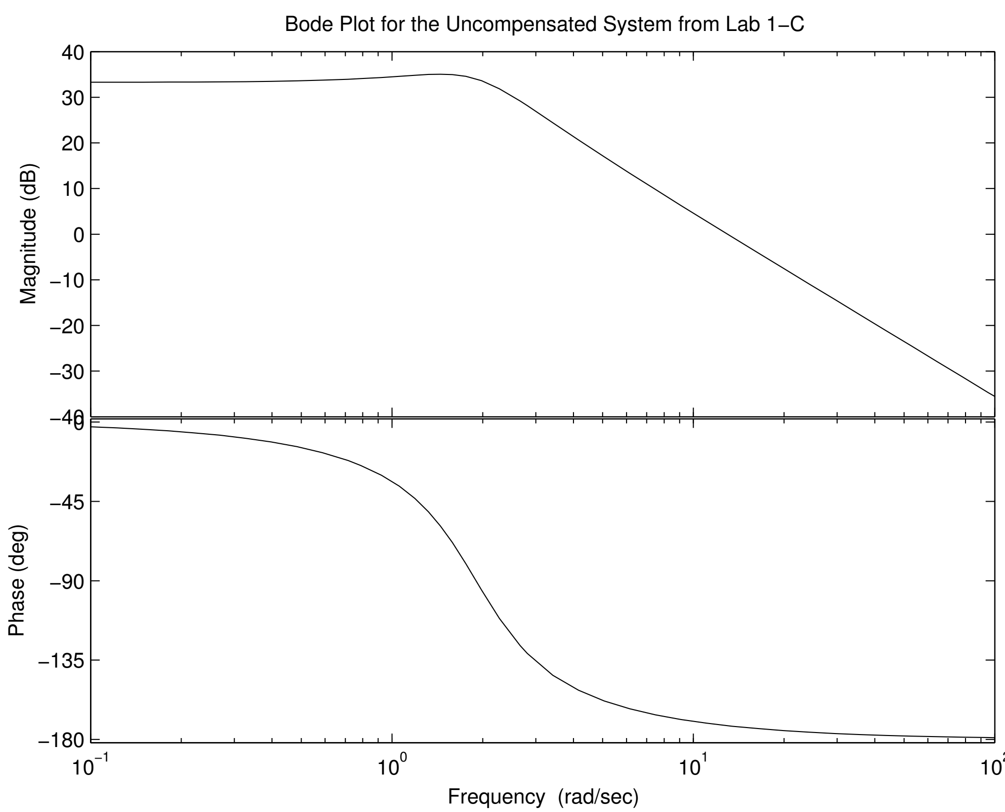
$$\frac{\theta_o}{V_{ref}} = \frac{-2G_cK_T}{6.75s(JRs + K_TK_e) + 2G_cK_TK_p}$$

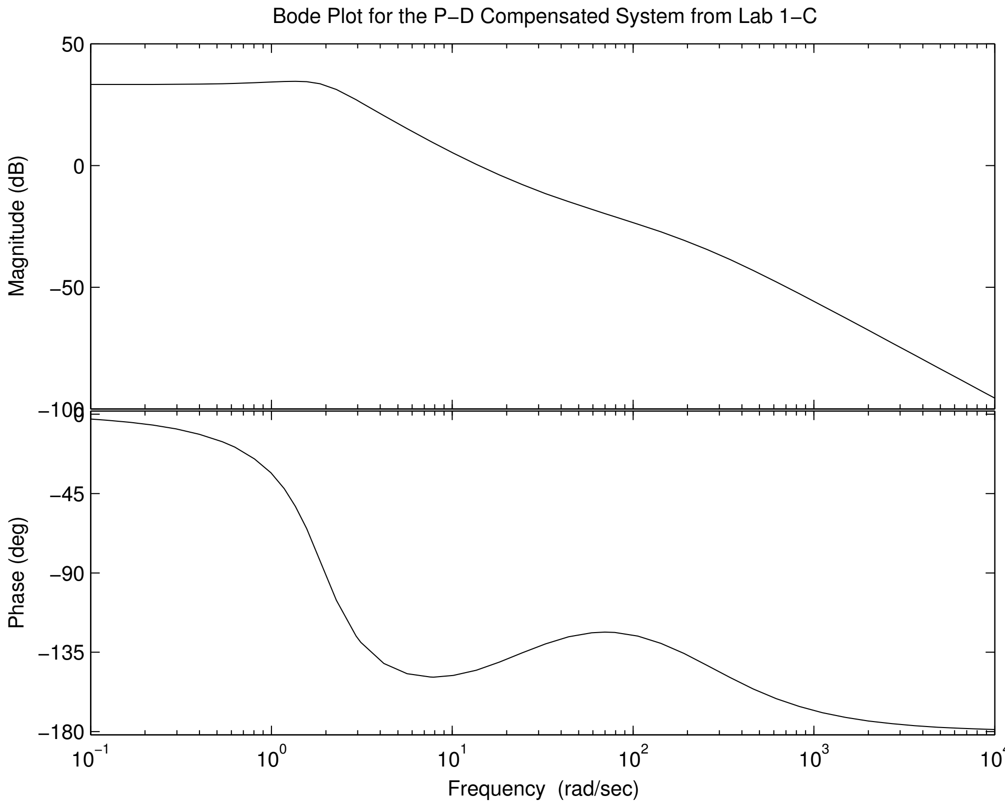
The closed loop transfer function for the lead compensated system is

$$\frac{\theta_o}{V_{ref}} = \frac{-2GK_T(\tau s + 1)}{6.75s(JRs + K_TK_e)(0.1\tau s + 1) + 2GK_TK_p(\tau s + 1)}$$

The uncompensated system has a phase margin of 20 degrees and a crossover frequency of 5 rad/sec.

The compensated system has a phase margin of 35 degrees and a crossover frequency of 14 rad/sec.

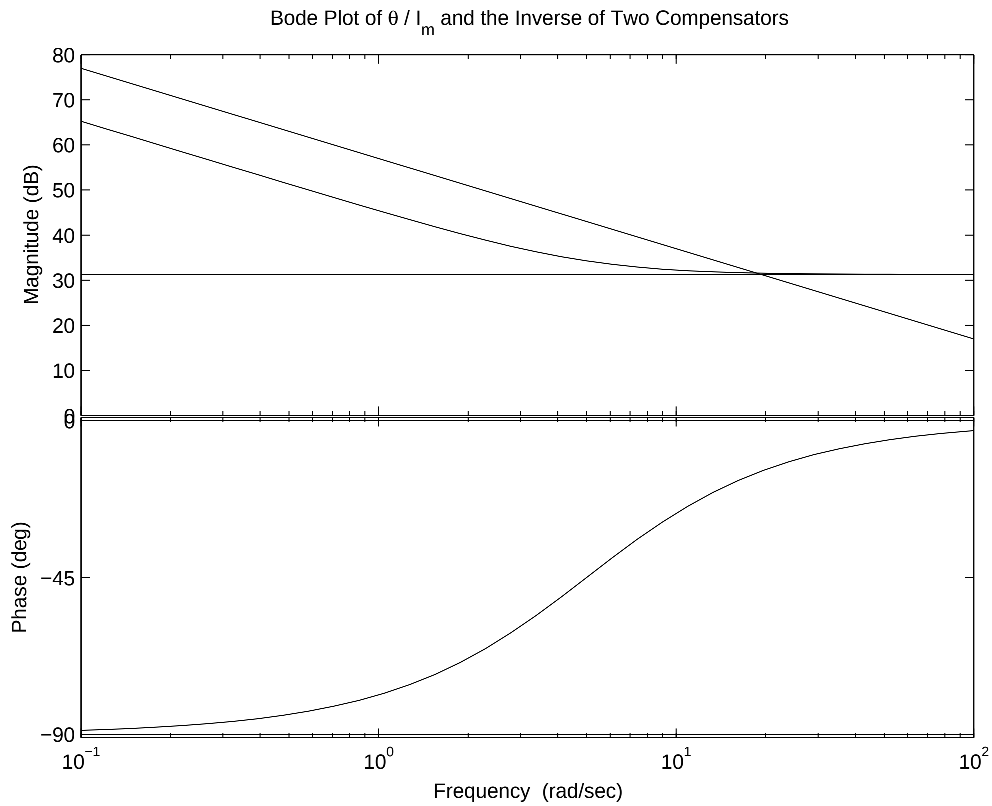




(d) Without minor loop compensation, the system includes a double integrator in the forward path, giving it -180 degrees of phase for all frequencies and making it unstable. The Bode plot below shows $\frac{\dot{\theta}}{I_m}$ and $\frac{1}{H_{min}}$ for two different choices of H_{min}. For minor loop compensation, the compensator magnitude is the lower curve of the minor loop forward path $\frac{\dot{\theta}}{I_m}$ and the inverse of the minor loop compensator $\frac{1}{H_{min}}$.

With minor loop compensation where $H_{min} = K_{tach}$, we expect to see constant gain in the inner loop for low frequencies until about $\omega = 10$ rad/sec when the magnitude rolls off at a rate of -20 dB per decade. Since the forward path of the outer loop includes an integrator, the system should crossover with a slope of 1 (assuming the gain elements are chosen propoerly), ensuring stability. However, the imminent arrival of the second pole means that there won't be much phase margin. Furthermore, since we only have one integrator in the forward path, we cannot offer zero steady state error to a ramp input. In other words, the low (finite) DC gain offered by the inner loop is insufficient to provide zero steady state error for ramp inputs to the whole system.

With minor loop compensation where $H_{min} = K_{tach}\frac{\tau s}{\tau s+1}$, we expect to see infinite DC gain in the inner loop until $\omega = 10$ rad/sec when the magnitude flattens out briefly and then rolls off at a rate of -20 dB per decade. Combined with the integrator in the outer loop, this meanse that the system loop gain looks like $\frac{1}{s^2}$ at low frequencies (including DC), but flattens out to a slope of -20 db per decade briefly before returning to a slope of -40 db per decade for high frequencies. The brief stretch of -20 dB per decade ensures that crossover occurs with plenty of phase margin assuing stability while the higher gain at low frequencies provides better tracking and lower steady state errors than the previous compensator.



(e) For $H_{min} = K_{tach}$, the outer loop gain is

$$L_o(s) = \frac{-G_1 K_p}{6.75 s K_{tach}(1 + s\frac{J}{0.4 G_2 K_T K_{tach}})}$$

which has a phase margin of 59 degrees and a crossover frequency of 18 rad/sec. For other $H_{min} = \frac{0.2 s K_{tach}}{0.2s+1}$, the outer loop gain is

$$L_o(s) = \frac{-G_1 K_p 0.4 G_2 K_T (\tau s + 1)}{6.75 s^2 (J\tau s + J + 0.4 G_2 K_T \tau K_{tach})}$$

which has a phase margin of 48 degrees and a crossover frequency of 17 rad/sec.